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Statistical interpretation of data —

Part 6: Determination of statistical tolerance intervals

Interprétation statistique des données —

Partie 6: Détermination des intervalles statistiques de tolérance

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 16269-6 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*.

This first edition of ISO 16269-6 cancels and replaces ISO 3207:1975, which has been technically revised.

ISO 16269 consists of the following parts, under the general title *Statistical interpretation of data*:

— *Part 6: Determination of statistical tolerance intervals*

— *Part 7: Median — Estimation and confidence intervals*

— *Part 8: Determination of prediction intervals*

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Introduction

A statistical tolerance interval is an estimated interval, based on a sample, which can be asserted with confidence $1 - \alpha$, for example 95 %, to contain at least a specified proportion p of the items in the population. The limits of a statistical tolerance interval are called statistical tolerance limits. The confidence level $1 - \alpha$ is the probability that a statistical tolerance interval constructed in the prescribed manner will contain at least a proportion p of the population. Conversely, the probability that this interval will contain less than the proportion p of the population is α. This part of ISO 16269 describes both one-sided and two-sided statistical tolerance intervals; a one-sided interval is constructed with an upper or a lower limit while a two-sided interval is constructed with both an upper and a lower limit.

Tolerance intervals are functions of the observations of the sample, i.e. statistics, and they will generally take different values for different samples. It is necessary that the observations be independent for the procedures provided in this part of ISO 16269 to be valid.

Two types of tolerance interval are provided in this part of ISO 16269, parametric and distribution-free. The parametric approach is based on the assumption that the characteristic being studied in the population has a normal distribution; hence the confidence that the calculated statistical tolerance interval contains at least a proportion p of the population can only be taken to be $1 - \alpha$ if the normality assumption is true. For normally distributed characteristics, the statistical tolerance interval is determined using one of the Forms A, B, C or D given in Annex A.

Parametric methods for distributions other than the normal are not considered in this part of ISO 16269. If departure from normality is suspected in the population, distribution-free statistical tolerance intervals may be constructed. The procedure for the determination of a statistical tolerance interval for any continuous distribution is provided in Forms E and F of Annex A.

The tolerance limits discussed in this part of ISO 16269 can be used to compare the natural capability of a process with one or two given specification limits, either an upper one U or a lower one L or both in statistical process management. An indication of this is the fact that these tolerance limits have also been called natural process limits. See ISO 3534-2:1993, 3.2.4, and the general remarks in ISO 3207 which will be cancelled and replaced by this part of ISO 16269.

Above the upper specification limit U there is the upper fraction nonconforming p_U (ISO 3534-2:—, 3.2.5.5 and 3.3.1.4) and below the lower specification limit L there is the lower fraction nonconforming p_L (ISO 3534-2:—, 3.2.5.6 and 3.3.1.5). The sum $p_U + p_L = p_T$ is called the total fraction nonconforming. (ISO 3534-2:—, 3.2.5.7). Between the specification limits U and L there is the fraction conforming $1 - p_T$.

In statistical process management the limits U and L are fixed in advance and the fractions p_U, p_L and p_T are either calculated, if the distribution is assumed to be known, or otherwise estimated. There are many applications of statistical tolerance intervals, although the above shows an example to a quality control problem. Wider applications and more statistical intervals are introduced in many textbooks such as Hahn and Meeker [10].

In contrast, for the tolerance intervals considered in this part of ISO 16269, the confidence level for the interval estimator and the proportion of the distribution within the interval (corresponding to the fraction conforming mentioned above) are fixed in advance, and the limits are estimated. These limits may be compared with U and L. Hence the appropriateness of the given specification limits U and L can be compared with the actual properties of the process. The one-sided tolerance intervals are used when only either the upper specification limit U or the lower specification limit L is relevant, while the two-sided intervals are used when both the upper and the lower specification limits are considered simultaneously.

The terminology with regard to these different limits and intervals has been confusing as the “specification limits” were earlier also called “tolerance limits” (see the terminology standard ISO 3534-2:1993, 1.4.3, where both these terms as well as the term “limiting values” were all used as synonyms for this concept). In the latest

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revision of ISO 3534-2:—, only the term specification limits have been kept for this concept. Furthermore, the *Guide for the expression of uncertainty in measurement* [5] uses the term “coverage factor” defined as a “numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty”. This use of “coverage” differs from the use of the term in this part of ISO 16269.

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Statistical interpretation of data —

Part 6:
Determination of statistical tolerance intervals

1 Scope

This part of ISO 16269 describes procedures for establishing tolerance intervals that include at least a specified proportion of the population with a specified confidence level. Both one-sided and two-sided statistical tolerance intervals are provided, a one-sided interval having either an upper or a lower limit while a two-sided interval has both upper and lower limits. Two methods are provided, a parametric method for the case where the characteristic being studied has a normal distribution and a distribution-free method for the case where nothing is known about the distribution except that it is continuous.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 3534-2:—[1)], *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms, definitions and symbols

3.1 Terms and definitions

For the purposes of this document, the terms and definition given in ISO 3534-1, ISO 3534-2 and the following apply.

3.1.1
statistical tolerance interval
interval determined from a random sample in such a way that one may have a specified level of confidence that the interval covers at least a specified proportion of the sampled population

NOTE The confidence level in this context is the long-run proportion of intervals constructed in this manner that will include at least the specified proportion of the sampled population.

3.1.2
statistical tolerance limit
statistic representing an end point of a statistical tolerance interval

NOTE Statistical tolerance intervals can be either one-sided, in which case they have either an upper or a lower statistical tolerance limit, or two-sided, in which case they have both.

1) To be published. (Revision of ISO 3534-2:1993)

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