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October 2002 Update
of the March 1979 Revision
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Welding Research Council bulletin

LOCAL STRESSES IN SPHERICAL
AND CYLINDRICAL SHELLS
DUE TO EXTERNAL LOADINGS

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These Bulletins contain final reports from projects sponsored by the Welding Research Council, important papers presented before engineering societies and other reports of interest.

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FOREWORD

October 2002 Update of March 1979 Revision

The October 2002 Update to the March 1979 Revision of WRC Bulletin 107 includes minor editorial changes for improvement and readability of several equations, curves and some text. There are NO technical changes.

- The calculation forms (Tables 2, 3 and 5) are improved, particularly to show the "+" and "-" quantities more definitively.
- The equation for stress in paragraph 3.6.3 is revised to be on one line.
- The parameter definitions on several of the curves (beginning with figure SR-1) are improved and clarified.
- Appendix B, exponents in Equations 1, 2, 3 and 4 are enlarged for readability.

PVRC thanks Mr. James R. Farr, Honorary Emeritus Member of the Pressure Vessel Research Council, for his assistance in preparing this update.

NOTE: WRC Bulletins 107 and 297 should be considered (and purchased) as an integral set. In addition, PVRC Technical Committees are working on a project that is envisioned to culminate in a new publication to add to the WRC Bulletin 107 and 297 set. The new publication will provide significant new technical information on local shell stresses from nozzles and attachments.

Greg L. Hollinger
The Pressure Vessel Research Council

FOREWORD

to March 1979 Revision

Welding Research Council Bulletin No. 107 has been one of the most widely used bulletins ever published by WRC. The original bulletin was published in August 1965. Since that time, a revised printing was issued in December 1968; a second revised printing was issued in July 1970; a third revised printing was released in April 1972; and a June 1977 reprint of the third revised printing was issued. As sometimes happens with publications of this type, some errors were detected and then corrected in subsequent revised printings.

In this March 1979 Revision of Bulletin 107, there are some additional revisions and clarifications. The formulations for calculation of the combined stress intensity, S , in Tables 2, 3, and 5 have been clarified. Changes in labels in Figures 1C-1, 2C-1, 3C, and 4C have been made and the calculated stresses for Model "R" in Table A-3 and Model "C-1" in Table A-4 have been revised accordingly. The background for the change in labels is given in a footnote on p. 66.

Present plans call for a review and possible extension of curves to parameters which will cover the majority of openings in nuclear containment vessels and large storage tanks. Plans are to extend R/T from 300 to 600 and to extend d/D range from 0.003 to 0.10 for the new R/T range, review available test data to establish limits of applicability, and develop some guidance for pad reinforcements.

Long range plans are to review shell theory in general, and Bijlaard's method in particular. The goal is to extend the R/t up to 1200 for a d/D up to 0.1. This will include large deflection theory and other nonlinear effects. In addition, available computer programs will be studied in hope of developing one which will be an appropriate supplement to Bijlaard's method. Finally, a review will be made of limit loads related to large R/T and small d/D .

J.R. Farr, *Chairman*
PVRC Design Division

FOREWORD

Several years ago, the Pressure Vessel Research Committee sponsored an analytical and experimental research program aimed at providing methods of determining the stresses in pressure vessel nozzle connections subjected to various forms of external loading. The analytical portion of this work was accomplished by Prof. P. P. Bijlaard of Cornell University, and was reported in References 1 to 8 inclusive. Development of the theoretical solutions involved a number of simplifying assumptions, including the use of shallow shell theory for spherical vessels and flexible loading surfaces for cylindrical vessels. These circumstances limited the potential usefulness of the results to d_i/D_i ratios of perhaps 0.33 in the case of spherical shells and 0.25 in the case of cylindrical shells. Since no data were available for the larger diameter ratios, Prof. Bijlaard later supplied data, at the urging of the design engineers, for the values of $\beta = 0.375$ and 0.50 (d_i/D_i ratios approaching 0.60) for cylindrical shells, as listed on page 12 of Reference 10. In so doing, Prof. Bijlaard included a specific warning concerning the possible limitations of these data, as follows: "The values for these large loading surfaces were computed on request of several companies. It should be remembered, however, that they actually apply to flexible loading surfaces and, for radial load, to the center of the loading surface. It should be understood that using these values for the edge of the attachment, as was recommended for small loading surfaces, may be unconservative."

Following completion of the theoretical work, experimental work was undertaken in an effort to verify the theory, the results of which were published in References 17 and 18. Whereas this work seemingly provided reasonable verification of the theory, it was limited to relatively small d_i/D_i ratios—0.10 in the case of spherical shells and 0.126 in the case of cylindrical shells. Since virtually no data, either analytical or experimental, were available covering the larger diameter ratios, the Bureau of Ships sponsored a limited investiga-

tion of this problem in spheres, aimed at a particular design problem, and the Pressure Vessel Research Committee undertook a somewhat similar investigation in cylinders. Results of this work have recently become available emphasizing the limitations in Bijlaard's data on cylindrical shells, particularly as it applies to thin shells over the "extended range" (page 12 of Reference 10).

Incident to the use of Bijlaard's data for design purposes, it has become apparent that design engineers sometimes have difficulty in interpreting or properly applying this work. As a result of such experience, PVRC has felt it desirable that all of Bijlaard's work be summarized in convenient, "cook-book" form to facilitate its use by design engineers. However, before this document could be issued, the above mentioned limitations became apparent, presenting an unfortunate dilemma, viz., the test data indicate that the calculated data are partially inadequate, but the exact nature and magnitude of the error is not known, nor is any better analytical treatment of the problem available (for cylinders).

Under these circumstances, it was decided that the best course was to proceed with issuing the "cook-book," extending Bijlaard's curves as best we can on the basis of available test data. This decision was based on the premise that all of the proposed changes would be toward the conservative (or "safe") side and that design engineers would continue to use Bijlaard's extended range data unless some alternative were offered. The following paper is therefore presented in the hope that it will facilitate the use of Bijlaard's work by design engineers. Every effort has been made to point out any known limitations in the work and to explain the exact nature of the changes which have been made to Bijlaard's original curves and data; however, users are warned that the resulting work is not necessarily adequate for all cases. It is the hope of the Subcommittee that additional theoretical work can be undertaken to provide more adequate data on various phases of this problem.

F. S. G. WILLIAMS, *Chairman*
PVRC Subcommittee on Reinforced
Openings and External Loadings

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Local Stresses in Spherical and Cylindrical Shells due to External Loadings

by K. R. Wichman, A. G. Hopper, and J. L. Mershon

1. Nomenclature

Symbols used in the formulas and equations in the text are listed below:

1.1 General Nomenclature

σ_i	= normal stress in the i th direction on the surface of the shell, psi
τ_{ij}	= shear stress on the i th face of the j th direction
S	= stress intensity = twice maximum shear stress, psi
N_i	= membrane force per unit length in the i th direction, lb/in.
M_i	= bending moment per unit length in the i th direction, in. lb/in.
K_n	= membrane stress concentration factor (pure tension or compression)
K_b	= bending stress concentration factor
i	= denotes direction. In the case of spherical shells, this will refer to the tangential and radial directions with respect to an axis normal to the shell through the center of the attachment as shown in Fig. 1. In the case of cylindrical shells, this will refer to longitudinal and circumferential directions with respect to the axis of the cylinder as shown in Fig. 2.
+	= denotes tensile stress (when associated with σ_i)
-	= denotes compressive stress (when associated with σ_i)
θ	= angle around attachment, degrees (see Figs. 1 and 2)
E	= modulus of elasticity, psi
P	= concentrated radial load or total distributed radial load, lb

V	= concentrated external shear load, lb
M	= concentrated external overturning moment, in. lb
M_T	= concentrated external torsional moment, in. lb
d_i, d_m	= inside diameter and mean diameter, respectively, of the nozzle, in.
D_i, D_m	= inside diameter and mean diameter, respectively, of the shell, in.

1.2 Nomenclature Applicable to Spherical Shells

V_1	= concentrated external shear load in 2-2 direction, lb
V_2	= concentrated external shear load in 1-1 direction, lb
M_1	= external overturning moment in 1-1 direction, in. lb
M_2	= external overturning moment in 2-2 direction, in. lb
R_m	= mean radius of spherical shell, in.
T	= thickness of spherical shell, in.
r_o	= outside radius of cylindrical attachment, in.
r_m	= mean radius of hollow cylindrical attachment, in.
t	= thickness of hollow cylindrical attachment, in.
τ	= r_m/t
ρ	= T/t
U	= $r_o/\sqrt{R_m T}$
N_r, N_θ	= membrane force in shell wall in radial and circumferential directions, respectively, lb/in. (see Fig. 1)
M_r, M_θ	= bending moment in shell wall in radial and circumferential directions, respectively, in. lb/in. (see Fig. 1)
σ_r	= normal stress in radial direction, psi (see Fig. 1)
σ_θ	= normal stress in circumferential direction, psi (see Fig. 1)

K.R. WICHMAN is with the Office of Standards Development of the U.S. Nuclear Regulatory Commission, Washington, DC. A.G. HOPPER, present location unknown. J.L. MERSHON, Consultant.

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